The Absolute Calibration of Periodic Microwave Phase Shifters Without a Standard Phase Shifter

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Abstract—An absolute calibration procedure for periodic microwave phase shifters is described. Increments on the phase shifter are compared by substitution with a reproducible phase-shift step of initially unknown value. The periodic nature of the phase shifter provides a reference phase shift of 360°, and alternative methods are outlined for the derivation of the correction curve from the step measurements. It is shown that the accuracy of the calibration is governed by the precision and reproducibility of the phase shifter under test.

I. Introduction

THIS PAPER shows how an accurate calibration of a cyclic microwave phase shifter of the Fox type [1] can be made using a simple experimental procedure and without the use of a primary phase standard; however, a computer is needed for data reduction.

Calibrations of phase shifters have been made in the past by complicated procedures using several similar phase shifters [2] or by recourse to a complex primary standard of extreme mechanical precision, requiring careful retuning for each new frequency [3]. In the latter method, the phase shift produced by the standard [4] is a function of the position of a movable part that has to be set, and its position read, with high accuracy. An excellent bibliography up to 1966 can be found in [5].

In the method to be described, advantage is taken of the periodic nature of the Fox-type phase shifter,[1] and, as will be shown, this type of instrument may be calibrated by this method with such accuracy that it may serve as a transfer standard to calibrate nonperiodic phase shifters.

II. Description of the Method

It was suggested by one of the authors (D. L. Hollway) that the calibration of a cyclic phase shifter of the Fox type could be carried out using a highly reproducible phase-shift step of an initially unknown value. Such a phase-shift step may be realized by inserting a dielectric flap through a central broad-wall slot into a waveguide to a precise depth. In further references this device will simply be called the "flap." Briefly, the principle of measurement is to record a set of phase-difference readings due to the insertion of the flap and to compute the correction curve from these differences, knowing that the readings correspond to points separated by an initially unknown but constant phase shift.

A. Measuring Setup

A bridge circuit is used having a sensitive phase-quadrature indication, which has been shown to be level insensitive to the first order [6].

The phase shifter under test (PSUT) and the flap are connected in tandem in the same arm of the bridge, are adequately isolated, and are operated between matched ports.[2]

The other arm of the bridge contains an auxiliary phase shifter used to set the initial balance before each measurement is taken. Bridge errors do not affect the results because each measurement is a direct substitution between the flap and the PSUT, the total phase shift in the measuring arm remaining constant.

Fig. 1 shows the bridge used in our measurements. This was a slightly modified version of the locating reflectometer [7], [8], operating in the band 8.2–12.4 GHz and set to display on an oscilloscope, the transmission coefficient of the measuring arm on an expanded Smith chart. The criterion for bridge balance was the crossing of the real axis by the spot, which moved close to the outer perimeter of the chart because of the low attenuation of the measuring arm. Final balance was indicated by a null detector having a sensitivity of $\pm 0.01°$ change in phase shift for full-scale indication. To utilize this sensitivity, the microwave frequency was phase locked to a harmonic of a temperature-stabilized crystal oscillator, and the measuring and reference arms of the bridge were made equal in length to minimize the effects of room-temperature variations. For example, at 10.6 GHz a phase shift of 0.01° corresponds to only 1-μm relative change in the lengths of the two bridge arms.

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[1] Strictly speaking, the Fox-type phase shifter is periodic in 720 electrical degrees, corresponding to one complete rotation of the tunnel.

A well-made phase shifter will also be periodic in a one-half rotation of the tunnel (360° on the scale). In practice, this has been found to be true. If not, then the cyclic interval referred to in this paper as 360 electrical degrees should be increased to 720 degrees.

[2] The flap need not necessarily operate between matched ports, provided that the phase indication is sufficiently insensitive to the level change caused by the variation in the mismatch loss when the flap is inserted.

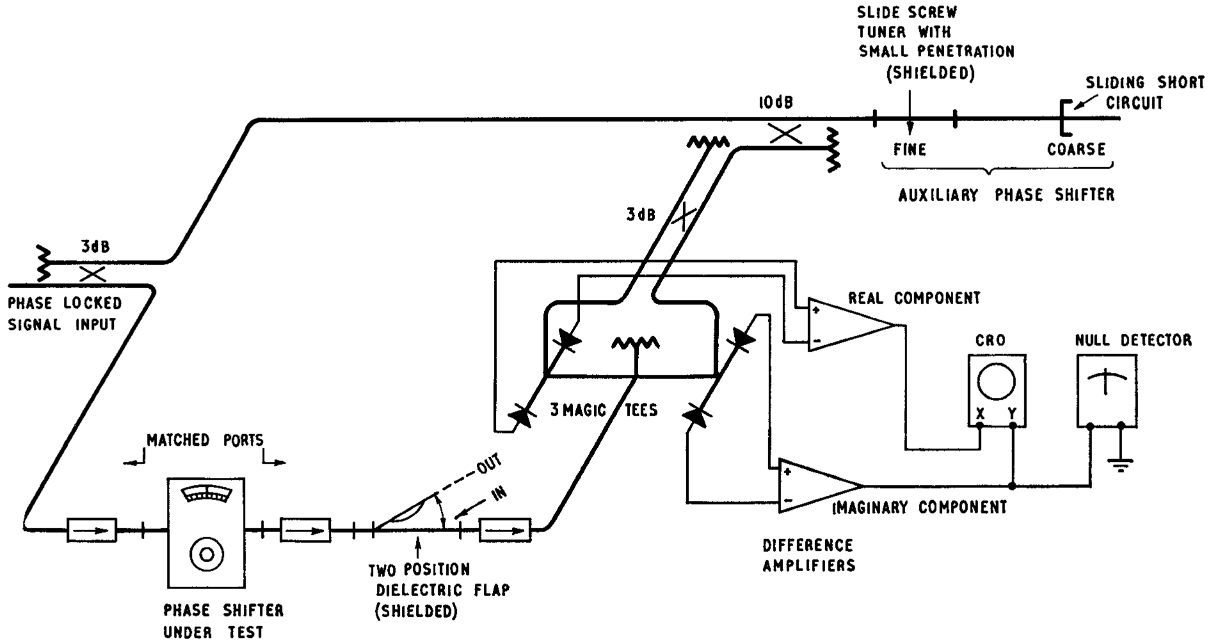

Fig. 1. Microwave phase bridge modified from the locating reflectometer [8].

B. Method of Measurement

In the method to be described, all measurements are made in pairs of readings of the PSUT. Each pair consists of an initial reading, denoted by P_i, made with the flap "in" and a second reading Q_i made after adjusting the PSUT to restore balance with the flap "out." The initial value of each pair is set exactly on a cardinal point by adjusting the auxiliary phase shifter, so providing a calibration at regular settings of the dial. The flap value may be chosen to lie anywhere in a broad optimum range, discussed in Section IV. For example, in a typical measurement, using a flap near 18.3°, the initial readings may be stepped at 10° intervals giving 36 pairs of values P_i, Q_i, e.g., 0°, 18.4°; 10°, 28.35°; 20°, 38.25°; $\cdots$ corresponding to the flap "in" and "out," respectively. The departures of $Q_i - P_i$ from the true flap value F are due to the errors of the PSUT. Our aim is to determine the correction curve of the PSUT from these strings of numbers.

In one variant, which we term the "quasi-buildup" method, the flap value is chosen to bring the second value of a pair quite close to the next initial value. This is of some advantage when the results are to be analyzed by the "interpolation" method, as will be described.

III. Methods of Computation

Two methods of computing the error curve from the string of pairs of readings have been used: Fourier analysis and interpolation. Both methods are capable of producing accurate correction curves. However, because the Fox-type phase shifter is an inherently cyclic instrument, the Fourier method has the advantage that, from the spectrum of the correction curve, it is often possible to identify and measure the contributions from different causes, as will be shown in Section V. Also, the Fourier method yields the calibration curve in an explicit form that may be used to evaluate the correction at any required point.

A. Interpolation Method

Let $C(P_i)$ and $C(Q_i)$ denote the corrections that must be added to the initial and final scale readings of the pair P_i and Q_i, and let the total number of pairs of readings be N, spaced so that the interval between successive initial readings is I $(=360/N$ degrees).

For every interval, the true phase change between P_i and Q_i is equal to the flap value F, or

$$Q_i + C(Q_i) - (P_i + C(P_i)) = F. \tag{1}$$

To derive a first approximation to the correction curve from these readings, it will be assumed that the correction curve is sufficiently smooth to allow us to relate $C(P_i)$ with $C(P_{i+1})$ by assuming that the rate of change of the correction in the interval P_i to P_{i+1} is equal to the rate of change in the range P_i to Q_i obtained from (1); thus

$$\frac{C(Q_i) - C(P_i)}{Q_i - P_i} = \frac{F}{Q_i - P_i} - 1 \simeq \frac{C(P_{i+1}) - C(P_i)}{I} \cdot \tag{2}$$

In the "conventional" buildup process used in other fields of measurement, it is usual to make Q_i closely equal to P_{i+1} and to assume that $C(Q_i) = C(P_{i+1})$. We have derived the more general procedure based on (2) because it is applicable even when the corrections are comparatively large, and because F need not be set closely equal to I.

It is convenient to specify the corrections with respect to that at the first point by letting $C(P_1)=0$; then the correction at any subsequent reading P_k is given by

$$C(P_k) = \sum_{i=1}^{k-1} (C(P_{i+1}) - C(P_i)) \simeq I \sum_{i=1}^{k-1} \left(\frac{F}{Q_i - P_i} - 1\right). \quad (3)$$

Because the phase shifter is cyclic, the correction curve closes on itself after 360° and therefore $C(P_{N+1}) = C(P_1)$. Using this and setting $k=N+1$ in (3) provides a close approximation to the flap value

$$F \simeq \frac{N}{\sum_{i=1}^{N} \frac{1}{Q_i - P_i}}. \quad (4)$$

The next step in the interpolation procedure is to use these values of $C(P_k)$ to obtain a related set of corrections at the Q points: $C(Q_k)$. The correction at each reading Q_k is obtained by using Stirling's formula to interpolate between the corrections at the five P readings closest to Q_k; for example, if $F=11°$ and $I=10°$, the closest five corrections would be $C(P_{k-1})$, $C(P_k)$, $\cdots$, $C(P_{k+3})$.

These computed corrections are then added to the appropriate P and Q readings to obtain improved sets of P' and Q' closer to the true phase shifts corresponding to the dial readings P and Q. This more accurate set of P', Q' is then fed back into (4) and (3) to start the next cycle of iteration. After each cycle, the flap value and corrections are tested by substitution in (1) and the iteration is stopped when the maximum discrepancy, for any value of i, is sufficiently small. Between two and eight iterations are usually required.

In practice, the convergence may be improved by changing the procedure so that the rate of change of correction in each interval, $(C(P_{i+1})-C(P_i))/I$, is computed, not directly from (2), but by interpolation between the nearest five known rates of change, viz., $(C(Q_j)-C(P_j))/(Q_j-P_j)$, where j takes integral values from $i-2$ to $i+2$.[3] Results computed by this method are shown in Fig. 3(a).

B. Fourier Analysis

For clarity in describing the method, let us assume that the correction curve, shown in Fig. 2, is a pure sine wave:

$$C(\alpha) = A_1 \sin \alpha \quad (5)$$

where α is the setting of the dial. For phase shifters of fair quality A_1 is 3° or less.

[3] Full details of these methods cannot be given here. We shall be pleased to send to readers additional information and copies of the computer programs, written in Fortran IV or in Basic.

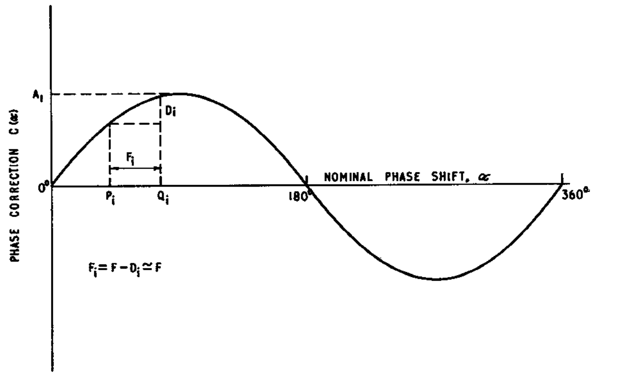

Fig. 2. Sinusoidal error curve of hypothetical phase shifter illustrating the measurement of an interval (Q_i-P_i) with a dielectric-flap phase step of F degrees.

To start the computation, an initial value of F is obtained from (4), which in practice gives a closer approximation to the true value than simply averaging all Q_i-P_i. The approximation is then made of neglecting the small corrections to be applied at the two dial settings to get the true phase shift, i.e., $Q_i=P_i+F$. This approximation may be used in the analysis without loss of accuracy because the results are corrected by an iterative procedure.

Substituting $Q_i=P_i+F$ into (5) we obtain

$$C(Q_i) = A_1 \sin (P_i + F) = A_1(\sin P_i \cos F + \cos P_i \sin F). \quad (6)$$

The apparent change in the flap value from pairs of P and Q is the difference between the corrections at Q_i and P_i, shown as D_i in Fig. 2. Combining (5) and (6) we obtain the following expression for D_i:

$$D_i = C(Q_i) - C(P_i) = 2A_1 \sin (F/2) \cos (P_i + F/2). \quad (7)$$

Thus for a PSUT having a sinusoidal correction curve, the "measured" curve would be a cosine with the same frequency, but of different amplitude and phase. In practice it is the measured curve that is available from the series P_i and Q_i, and if the flap F is known, we may derive the correction curve of the instrument. Let us assume a measured curve given by

$$Q_i - P_i - F = D_i = C(Q_i) - C(P_i) = A \cos (P_i + \phi). \quad (8)$$

Since (8) is analogous to (7), by comparing (7) with (5) we may write the expression analogous to (5), which is the correction curve in terms of the measured curve:

$$C(\alpha) = \frac{A}{2 \sin (F/2)} \sin (\alpha + \phi - F/2). \quad (9)$$

When carrying out an actual measurement, the measured curve D_i ($i=1$ to N) will not be a pure sinusoid; however, a Fourier analysis will reveal its constituent sinusoids (after the cosine and sine pairs have been combined into a single cosine curve), and (9) may be applied to each harmonic of the measured curve in turn, yielding the corresponding harmonics of the first approximation to the correction curve. Thus

$$C_m(\alpha) = \frac{A_m}{2 \sin (mF/2)} \sin (\alpha + \phi_m - mF/2). \quad (10)$$

Note that F has to be multiplied by the harmonic number m because the period of the mth harmonic is m times shorter than that of the fundamental component of the correction curve.

The instrument's correction curve is then synthetized from the Fourier components (10) in the usual way:

$$C(\alpha) = \sum_{m=1}^{h} C_m(\alpha) \quad (11)$$

where h is the highest permissible harmonic number, which is half the number of the measured pairs of points, i.e., $h = N/2$. The harmonic of order h is not calculable fully, since for this harmonic only two points are available per wave. In practice, the number of harmonics to be used in the calibration must be decided initially and N must be made large enough to define these harmonics fully.

From this approximation to the correction curve $C(\alpha)$, a more accurate value for the flap is computed in the following way. An improved value of phase shift is computed at all the measured points by adding the values of the corrections to these, and the differences between the corrected measured points,

$$F_i = Q_i + C(Q_i) - (P_i + C(P_i)) \quad (12)$$

are used to compute a more accurate flap value by inverse averaging F_i according to (4), as before. If the computed correction curve were a perfect description of the errors of the PSUT, all the F_i values of (12) would be the same. Because of the approximation made in the computation, F_i will vary with i, but to a lesser extent, and these variations represent the inaccuracies of the correction curve just computed.

The departures of F_i from the newly computed flap value may be regarded, for the purpose of computation, as a measurement, since our original measurement consisted of noting departures from a constant flap value caused by the errors of the PSUT. Thus the series $D_i = F_i - F$ may be fed back to the beginning of the program, and a correction curve to the previous correction curve may be computed and then added to it to obtain an improved correction curve to the PSUT. The largest (absolute) value of D_i, termed the "maximum inconsistency" $D_{i\max}$, is noted and the iteration is stopped when $D_{i\max}$ is not smaller than its previous value. It has been found in practice that about five iterations are needed to obtain the final correction curve. After each, $D_{i\max}$ is printed out together with i, the index of the reading pair at which the maximum occurs. If i stays constant for the first few iterations, for cases where the highest harmonic order used is less than $N/2$, it means either that the instrument has a local anomaly on its dial near the ith pair of readings, which is hard to fit in a least-squares sense,[4] or that either P_i or Q_i has been misread. A virtue of the method is that the suspected points can be remeasured later without having to repeat the whole calibration.

After completing the final calculation of the correction curve, the result is plotted, tabulated, and its Fourier components are printed out. This Fourier spectrum may reveal hidden characteristics of the PSUT, as will be shown in Section VI.

It has been pointed out by G. W. Small of the National Standards Laboratory [9] that if a reasonable assumption is made about the harmonic of order h $(=N/2)$, one phase of this harmonic may be assigned a zero value and the N equations can then be solved for the flap value and the remaining $N-1$ coefficients of the harmonic components of the correction curve. A comparison made between this and the iterative procedure described above showed a negligible difference between the results obtained by the two methods.

IV. Choice of Flap Value

The flap value is not critical and without loss of accuracy may be set anywhere between upper and lower bounds. At the low end this range is limited chiefly by the dial resolution of the PSUT. After investigating typical cases using an artificial model for the PSUT, it was found, for instance, that for a PSUT having a maximum error of 0.5°, a flap value smaller than 10° should not be used unless the dial resolution is better than 0.1°.

An upper limit for the flap value is set by the fact that the measurement is not capable of observing a harmonic component of the correction curve that is periodic in flap length.

It has been found that for calibrating phase shifters of quality presently available, a good compromise between accuracy and time taken to complete a measurement is obtained by taking 36 or 40 pairs of points, corresponding to stepping intervals of 10° or 9°, respectively. In these cases, the respective highest computable harmonics are the 18th and the 20th; thus the maximum flap lengths should be under 20° or 18°.

V. Accuracy

Both the Fourier and the interpolation methods have been tested extensively using artificial correction curves made up either of combinations of sinusoids or of triangular or other pulses. For the sinusoidal form,

$$C(\alpha) = A_1 \sin (m_1\alpha + \phi_1) + A_2 \sin (m_2\alpha + \phi_2) + \cdots + A_N \sin (m_N\alpha + \phi_N). \quad (13)$$

A simulated measurement is carried out by stipulating a flap value F and interval I and using a computer to find every Q_i, the reading at which the true phase shift differs from that at P_i by F, according to (1). As $C(\alpha)$ is given by (13), (1) becomes transcendental and must be solved by iteration for Q_i.

[4] It is a property of Fourier analysis to fit a set of points by each harmonic in a least-squares sense.

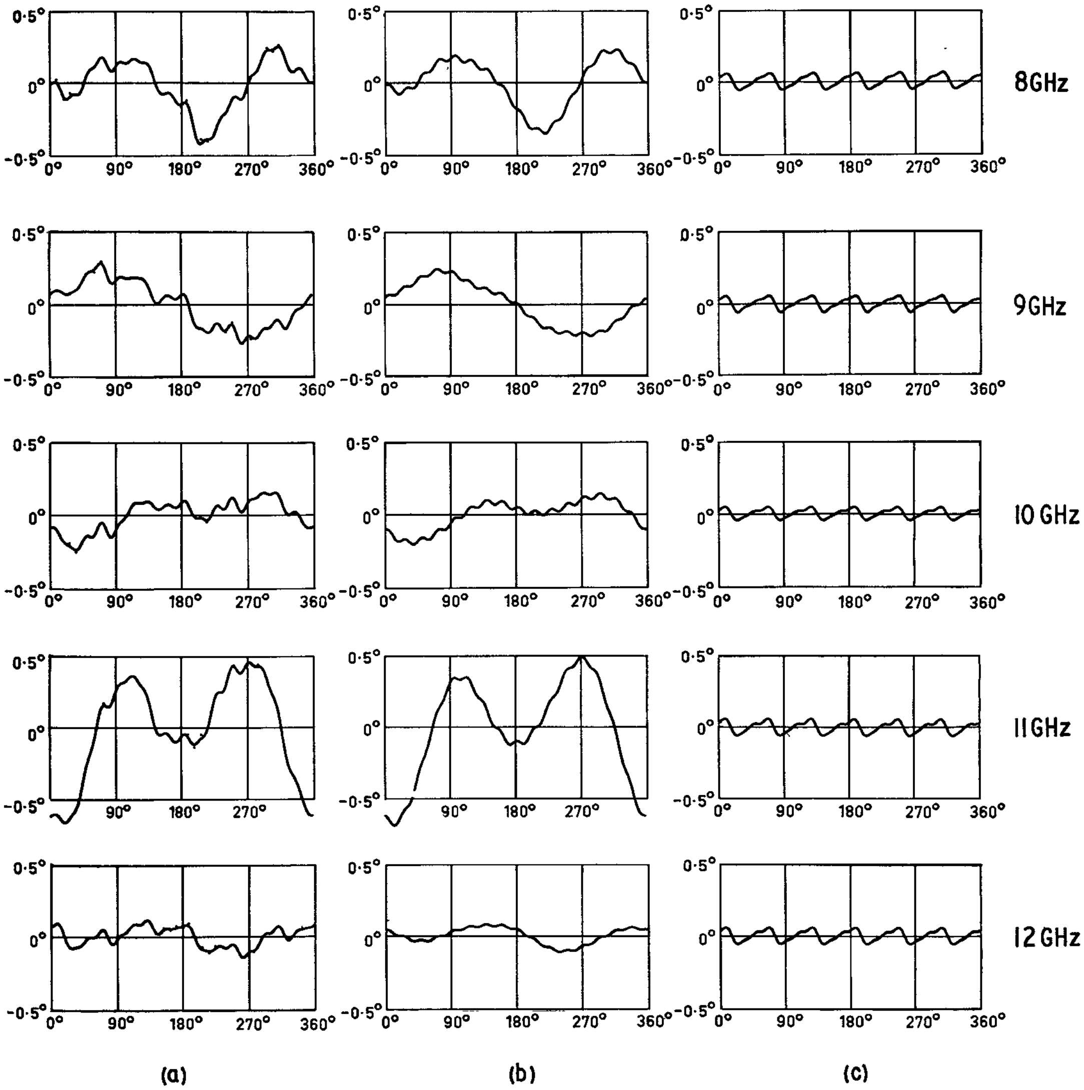

Fig. 3. Correction curves for a rotary phase shifter measured at five frequencies and computed by the Fourier method. (a) Complete correction curves computed both by the Fourier method using the first 20 harmonics and by the interpolation method. Points by the interpolation method, which differed by more than 0.01° from the Fourier result, are shown individually. (b) "Electrical" errors; the resultant of the 1st, 2nd, 3rd, 4th, and 16th harmonics. (c) "Mechanical" errors; the resultant of the 6th, 12th, and 18th harmonics. Note that these components are independent of frequency in both amplitude and phase.

These values of P_i and Q_i were then regarded as "measured data" and a correction curve was computed. Inspection of the Fourier components of this curve revealed that the "unwanted" components [i.e., those absent from the original correction curve (13)] were at least 10^4 times smaller than the "genuine" components.

The maximum inconsistency $D_{i\max}$ (see Section III-B) was also used to assess the goodness of fit. If harmonics up the highest possible order are used, a "perfect-fit" solution results, with $D_{i\max}$ tending to zero. (Using a small computer with a 23-bit fraction, the final value of $D_{i\max}$ is about 3.10^{-6} degrees.)

In these tests, the computed flap value was, on the average, within 0.0001° of the known stipulated value. This proves that when an actual phase shifter is calibrated by this method, the overall accuracy is governed by the resolution and reproducibility of the phase shifter itself, which for presently available phase shifters is about 0.05°.

In a typical test of the interpolation method, a correction curve was used consisting of a triangular pulse 90° wide and 3° high. For $I=10°$ and $F=10.5°$, this curve was reproduced after two iterations with a maximum error of 0.010° and $D_{i\max}=0.001°$. The flap value was found correctly to within 0.0002°. After four iterations, $D_{i\max}$ had fallen to 0.0001°, but the maximum error increased slightly to 0.011°. The flap value did not change.

In the foregoing it has been assumed that the PSUT is fully isolated and is operated between matched ports. Additional errors will be introduced if the ports are mismatched, but this limitation is common to all methods of phase-shifter calibration and the limits of error are readily calculable.

VI. Results

The results of the calibration of a modified commercial X-band rotary phase shifter are shown in Fig. 3. The Fourier spectrum of the computed correction curve at 8 GHz is shown in Fig. 4. At all the five frequencies

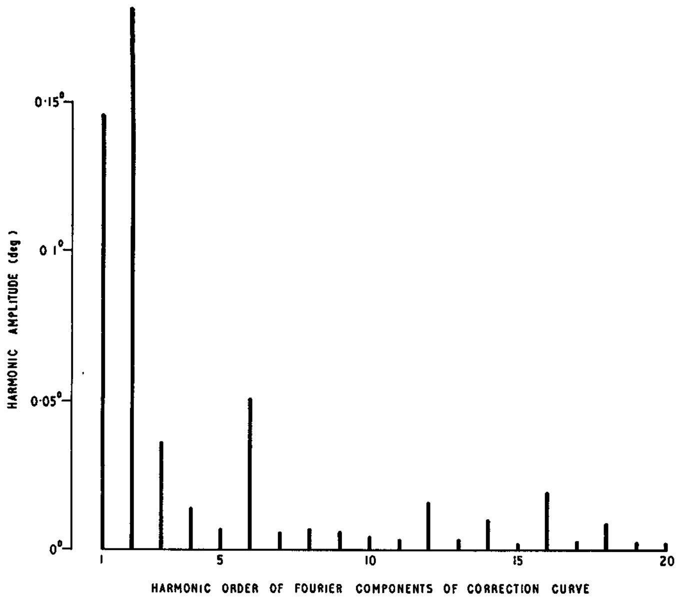

Fig. 4. Fourier spectrum of the computed correction curve at 8 GHz, shown in Fig. 3(a).

of measurement the dominant components were found to be the 1st, 2nd, 3rd, 4th, 6th, 12th, 16th, and 18th harmonics. Because this phase shifter was constructed with a reduction gear requiring six rotations of the dial for a 360° phase shift, it was suspected that the presence of the 6th harmonic and its multiples was due to mechanical errors. Closer inspection of the five Fourier spectra revealed that the 6th harmonic and its multiples are independent of frequency, both in amplitude and phase [see Fig. 3(c)], proving their mechanical origin.

In Fig. 3(b) the resultant curves of the 1st, 2nd, 3rd, 4th, and 16th harmonics are shown, labeled "electrical" errors, which are frequency dependent. The most likely causes of the low-order components of the "electrical" error are imperfect conversion to and from circular polarization by the quarter-wave sections and internal reflections. For example, two end reflections with amplitudes of 0.04 would cause a second harmonic of the error spectrum of about 0.1°. The presence of the 16th harmonic is interesting. It was found at all five frequencies of measurement with an almost constant amplitude, but with a slowly varying phase. It is believed that this error component is caused by $TE_{16,n}$ evanescent modes, which are preferentially set up by two diametrically opposed 0.1-in wide slots in the 1-in diameter rotating section, required for broad-banding.

To show that the computed correction curve is characteristic of the instrument and not of the method of computation, Fig. 3(a) displays the results of computations by both the Fourier and the interpolation methods. Points by the interpolation method that differed by more than 0.01° from the Fourier result are shown individually.

VII. Conclusion

A new method of calibrating microwave rotary phase shifters has been described, which does not require the use of a standard phase shifter. Instead, a highly repeatable phase step, realized as a dielectric flap, is used to measure differences of nominal phase shift spaced around the dial of the phase shifter under test. This method has the advantage that the calibration may be carried out at any frequency without the need for prior tuning of a phase standard. All that is needed when changing frequency is to rematch the ports facing the phase shifter under test. No knowledge of λ_g is necessary to complete a calibration.

Two methods of computing the calibration curve have been presented. It has been shown that the Fourier method is able to separate the "mechanical" from the "electrical" errors and it gives the calibration curve of the instrument in an explicit form. Thus the calibration is available at any point without interpolation. The measurement procedure is simple and repetitive and is insensitive to long-term drifts.

The method described results in calibration curves of high accuracy and, so far, of unparalleled detail.

Acknowledgment

The authors wish to thank P. W. Campbell, who constructed the only special component needed: the precisely reproducible dielectric-flap phase step.

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